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**Theorem 1.20.**

$$\nabla \cdot \frac{\hat{r}}{r^2} = 4\pi\delta(\mathbf{r}).$$

*Proof.* Since  $r$  is defined as  $r = |\mathbf{x} - \mathbf{x}'|$ , first choose a coordinate system in which  $\mathbf{x}'$  is at the origin. This will not affect the equality (why?), but this will allow us to use spherical coordinates. First let's assume that  $r \neq 0$ . Evaluating the divergence in spherical coordinates,

$$\nabla \cdot \frac{\hat{r}}{r^2} \stackrel{1}{=} \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{1}{r^2} = \frac{1}{r^2} \frac{\partial}{\partial r} (1) = 0.$$

Therefore the divergence is zero everywhere except possibly at the origin. To check the divergence at the origin, we will perform an integral of the divergence over a sphere of radius  $R$  centered around the origin and use the Divergence Theorem.

$$\begin{aligned} \int_V \nabla \cdot \frac{\hat{r}}{r^2} d^3x &\stackrel{2}{=} \int_S \frac{\hat{r}}{r^2} \cdot \hat{n} da = \int_S \frac{\hat{r} \cdot \hat{n}}{r^2} da \\ &= \int_0^{2\pi} \int_0^\pi \frac{1}{R^2} R^2 \sin(\theta) d\theta d\phi = 4\pi. \end{aligned}$$

Therefore the divergence is zero everywhere except at the origin and any integral of the divergence containing the origin yields  $4\pi$ , so  $\nabla \cdot \frac{\hat{r}}{r^2} = 4\pi\delta(\mathbf{r})$ .  $\square$

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